Casimir effect for scalar fields with Robin boundary conditions in Schwarzschild background

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Abstract

The stress tensor of a massless scalar field satisfying Robin boundary conditions on two one-dimensional wall in two-dimensional Schwarzschild background is calculated. We show that vacuum expectation value of stress tensor can be obtained explicitly by Casimir effect, trace anomaly and Hawking radiation.

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1 Introduction

The Casimir effect is one of the most interesting manifestations of non-trivial properties of the vacuum state in quantum field theory [1, 2]. Since its first prediction by Casimir in 1948 [3], this effect has been investigated for various cases of boundary geometries and various types of fields [4]-[8]. The Casimir effect can be viewed as a polarization of vacuum by boundary conditions. Another type of vacuum polarization arises in the case of an external gravitational field. In this paper we study a situation when both types of sources for the polarization are present. There are several methods to calculate Casimir energy. For instance, we can mention mode summation, Green's function method [1], heat kernel method [6]along with appropriate regularization schemes such as point separation [9],[10] dimensional regularization [11], zeta function regularization [12, 13, 4, 5]. Recently a general new methods to compute renormalized one–loop quantum energies and energy densities are given in [14, 15] (see also [16]).

It has been shown [17, 18] that particle creation by a black hole in four dimensions is a consequence of the Casimir effect for a spherical shell. It has been shown that just the existence of the horizon and of the barrier in the effective potential are sufficient to compel the black hole to emit black-body radiation with a temperature that exactly coincides with the standard result for Hawking radiation. In [18], the results for the accelerated mirror have been used to prove the above statement.

The renormalized vacuum expectation value of the stress tensor of the scalar field in the Schwarzschild spacetime can be obtained by using different regularization methods.(see Refs. [19]-[26]). $< T^{\mu}_{\nu} >_{ren}$ is needed , for instance, when we want to study back-reaction, i.e, the influence that the matter field in a curved background assert on the background geometry itself. This would be done by solving the Einstein equations with the expectation value of the energy-momentum tensor as source.

In this paper the Casimir energy for massless scalar field in two- dimensional Schwarzschild black hole for two parallel plates with Robin boundary conditions is calculated. The Casimir energy is obtained by imposing general requirements. Calculation of the renormalized stress tensor for massless scalar field in two-dimensional Schwarzschild background has been done in [19, 27, 28]. The Casimir effect for a massless scalar filed under Dirichlet boundary condition in a two-dimensional Domain wall, Schwarzschild black hole, stringy black hole, Achucarro-Ortiz black hole, have been studied respectively in [29, 30, 25, 31, 32]. The Robin boundary condition includes the Dirichlet and Neumann boundary conditions as special cases. The Casimir effect for the general Robin boundary conditions on background of the Minkowski spacetime was investigated in Ref. [33] for flat boundaries, here we use the results of this reference to generate vacuum energy-momentum tensor in our interesting background. Knowing the Casimir energy in flat space and the trace anomaly can help us to calculate renormalized stress tensor. In other situations, besides the two previous quantities, we use Hawking radiation, which also has a contribution in the stress tensor. Therefore, the renormalized stress tensor is not unique and depends on the vacuum under consideration. Our paper is organised as follows. In section 2 general properties of stress tensor are discussed. Then in section 3, the vacuum expectation value of the stress tensor in two dimensions is obtained. Finally, in section 4, we conclude and summarize the results.

2 General properties of stress tensor

In a semiclassical framework for yielding a sensible theory of back reaction, Wald [34] has developed an axiomatic approach. There one tries to obtain an expression for the renormalized $T_{\mu\nu}$ from the properties (axioms) which it must fulfill. The axioms for the renormalized energy momentum tensor are as follows.

1-For off-diagonal elements, the standard result should be obtained.

2-In Minkowski spacetime, the standard result should be obtained.

3-Expectation values of energy-momentum are conserved.

4-Causality holds .

5-The energy-momentum tensor contains no local curvature tensor depending on derivatives of the metric higher than second order.

Two prescriptions that satisfy the first four axioms can differ by at most a conserved local curvature term. Wald [35], showed any prescription for renormalized $T_{\mu\nu}$ which is consistent with axioms 1-4 must yield the given trace up to the addition of the trace of conserved local curvature. It must be noted that trace anomalies in a stress tensor, i.e. the non-vanishing T^{μ}_{μ} for a conformally invariant field after renormalization originate from some quantum behavior [36]. In two-dimensional spacetime one can show that a trace-free stress tensor cannot be consistent with conservation and causality if particle creation occurs. A trace-free, conserved stress tensor in two dimensions must always remain zero if it is initially zero. One can show that the 'Davies-Fulling-Unruh' formula [37] for the stress tensor of a scalar field which yields an anomalous trace, $T^{\mu}_{\mu} = \frac{R}{24\pi}$, is unique which is consistent with the above axioms. In four dimensions, just as in two dimensions, a trace-free stress tensor which agrees with the formal expression for the matrix elements between orthogonal states cannot be compatible with both conservation laws and causality . It must be noted that, as Wald showed [35], with Hadamard regularization in the massless case axiom (5) cannot be satisfied unless we introduce a new fundamental length scale for nature. Regarding all of these axioms, thus we are able to obtain an unambiguous prescription for calculating the stress tensor.

3 Vacuum expectation values of stress tensor and Scalar Casimir effect

Our background shows a Schwarzschild black hole with the following metric:

$$d^2s = -(1-2\frac{m}{r})dt^2 + (1-2\frac{m}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2 d\varphi^2). \tag{1}$$

Now we reduce the dimension of spacetime to two,

$$d^2s = -(1-\frac{2m}{r})dt^2 + (1-\frac{2m}{r})^{-1}dr^2. \tag{2}$$

The metric (2) can be written in conformal form

$$d^2s = \Omega(r)(-dt^2 + dr^{*2}), \tag{3}$$

with

$$\Omega(r) = 1-\frac{2m}{r}, \qquad \frac{dr}{dr^*} = \Omega(r). \tag{4}$$

From now on, our main goal is to determine a general form of conserved energy-momentum tensor with, regard to the trace anomaly for the metric (2). It is necessary we mention that our method here is available for any general two-dimensional spacetime as following metric easily (see also [25, 38], [29]-[32])

$$d^2s = -g(r)dt^2 + g(r)^{-1}dr^2. \tag{5}$$

For the non-zero Christoffel symbols of the metric (2), we have in (t, r^*) coordinates:

$$\Gamma^{r^*}_{tt} = \Gamma^{t}_{tr^*} = \Gamma^{t}_{r^*r^*} = \Gamma^{r^*}_{r^*r^*} = \frac{1}{2}\frac{d\Omega(r)}{dr}. \tag{6}$$

Then the conservation equation takes the following form:

$$\partial_{r^*} T^{r^*}_t + \Gamma^t_{tr^*} T^{r^*}_t - \Gamma^{r^*}_{tt} T^t_{r^*} = 0, \tag{7}$$

$$\partial_{r^*} T^{r^*}_{r^*} + \Gamma^t_{tr^*} T^{r^*}_{r^*} - \Gamma^t_{tr^*} T^t_t = 0 \tag{8}$$

in which,

$$T^t_{r^*} = -T^{r^*}_t, \qquad T^t_t = T^\alpha_\alpha - T^{r^*}_{r^*} \tag{9}$$

and T^α_α is the anomalous trace in two dimension. Using equations (6),(9)we can rewrite Eq.(7) as following

$$\frac{d}{dr^*}T^{r^*}_t + \frac{1}{2}\frac{d\Omega}{dr}T^{r^*}_t + \frac{1}{2}\frac{d\Omega}{dr}T^{r^*}_t = \frac{d}{dr}T^{r^*}_t\frac{dr^*}{dr} + \frac{d\Omega}{dr}T^{r^*}_t = 0, \tag{10}$$

now using Eq.(4)we have

$$\frac{d}{dr}(\Omega(r)T^{r^*}_t) = 0. \tag{11}$$

Similarly for Eq.(8) we have

$$\frac{d}{dr^*}T^{r^*}_{r^*} + \frac{1}{2}\frac{d\Omega}{dr}T^{r^*}_{r^*} - \frac{1}{2}\frac{d\Omega}{dr}(T^\alpha_\alpha - T^{r^*}_{r^*}) = 0, \tag{12}$$

again using Eq.(4)we have

$$\frac{d}{dr}(\Omega(r)T^{r^*}{}_{r^*}) = \frac{1}{2}(\frac{d\Omega(r)}{dr})T^\alpha_\alpha. \tag{13}$$

Then Eq.(11) leads to

$$T^{r^*}_t = \alpha\Omega^{-1}(r), \tag{14}$$

where α is a constant of integration. The solution of Eq.(13) may be written in the following form:

$$T^{r^*}_{r^*}(r) = (H(r) + \beta)\Omega^{-1}(r), \tag{15}$$

where

$$H(r) = 1/2\int_l^r T^\alpha_\alpha(r')\frac{d}{dr'}\Omega(r')dr', \tag{16}$$

with l being an arbitrary scale of length and considering

$$T^\alpha_\alpha = \frac{R}{24\pi} = \frac{m}{6\pi r^3} \tag{17}$$

the function $H(r)$ produces the non-local contribution of the trace $T^{\alpha}_{\alpha}(x)$ to the energy-momentum tensor. Finding l depends on the metric. For the metric (2) we choose [8]

$$l \approx r_b = 2m, \tag{18}$$

so we reach

$$H(r) = \frac{m^2}{24\pi}\left(\frac{1}{16m^4} - \frac{1}{r^4}\right). \tag{19}$$

Using Eqs. (9,14)and (15) it can be shown that the energy-momentum tensor takes the following form in (t, r^*) coordinates. So we have the most general form of stress tensor field in our background of interest:

$$T^{\mu}{}_{\nu}(r) = \begin{pmatrix} T^{\alpha}_{\alpha} - \Omega(r)^{-1}H(r) & 0 \\ 0 & \Omega(r)^{-1}H(r) \end{pmatrix} + \Omega^{-1}\begin{pmatrix} -\beta & -\alpha \\ \alpha & \beta \end{pmatrix}. \tag{20}$$

Now we are going to obtain two constants α and β by imposing the second axiom of the renormalization scheme. We consider two one-dimensional walls which are placed at point r_1 and r_2 in our interest background. The massless scalar field whose energy-momentum tensor we try to evaluate satisfies the Robin boundary conditions on the walls. At first we review the casimir effect for massless scalar filed under Robin boundary condition on plates in Minkowski spacetime briefly (see [33]), We will assume that the field satisfies the mixed boundary condition

$$(a_j + b_j n^{\mu}\nabla_{\mu})\,\varphi(x) = 0, \quad r = r_j, \quad j = 1, 2 \tag{21}$$

on the plate $r = r_1$ and $r = r_2$, $r_1 < r_2$, n^{μ} is the normal to these surfaces, $n_{\mu}n^{\mu} = -1$, and a_j, b_j are constants. The results in the following will depend on the ratio of these coefficients only. However, to keep the transition to the Dirichlet and Neumann cases transparent we will use the form (21). In the case of a conformally coupled scalar the corresponding regularized VEV's for the energy-momentum tensor are uniform in the region between the plates and have the form

$$\langle T^{\mu}_{\nu}[\eta_{\alpha\beta}]\rangle_{\rm ren} = -\frac{J_1(B_1, B_2)}{2\pi^{1/2}a^2\Gamma(3/2)}\mathrm{diag}(1, -1), \quad r_1 < r < r_2, \tag{22}$$

where

$$J_1(B_1, B_2) = \text{p.v.}\int_0^{\infty} \frac{t\,dt}{\frac{(B_1t-1)(B_2t-1)}{(B_1t+1)(B_2t+1)}e^{2t} - 1}, \tag{23}$$

and we use the notations

$$B_j = \frac{\bar{b}_j}{\bar{a}_j a}, \quad j = 1, 2, \quad a = r_2 - r_1. \tag{24}$$

For the Dirichlet scalar $B_1 = B_2 = 0$ and one has $J_D(0,0) = 2^{-2}\Gamma(2)\zeta_R(2)$, with the Riemann zeta function $\zeta_R(s)$. Note that in the regions $r < r_1$ and $r > r_2$ the Casimir densities vanish :

$$\langle \bar{T}^{\mu}_{\nu}[\eta_{\alpha\beta}]\rangle_{\rm ren} = 0, \quad r < r_1, r > r_2. \tag{25}$$

The two-dimensional Schwarzschild spacetime is asymptotically flat,i.e at infinity is Minkowski spacetime, so the constants of integration α and β are evaluated demanding the regularized stress-tensor Eq.(20) to coincide with the standard Casimir stress tensor Eq.(22) at

infinity $r \to \infty$. Here we introduce the state vector $|C>$, which is the analogue of the Boulware state [39]. In the case of the existence of a boundary the Minkowski limit of $|C>$ is not the Minkowski state $|M>$. In this limit $<C|T_{\mu\nu}|C>$ is non-zero and shows the effects of the boundary conditions on the vacuum of the scalar filed, therefore we obtain

$$\beta = \varepsilon_c^1 - \frac{1}{384\pi m^2}, \qquad \alpha = 0. \tag{26}$$

where ε_c^1 is given by

$$\varepsilon_c^1 = -\frac{J_1(B_1, B_2)}{2\pi^{1/2}a^2\Gamma(3/2)}. \tag{27}$$

Now we consider the Hartle-Hawking state $|H>$ [40]. This state is not empty at infinity,even in the absence of boundary conditions on the quantum filed, but it corresponds to a thermal distribution of quanta at the Hawking temperature $T = \frac{1}{8\pi m}$. In fact, the state $|H>$ is related to a black hole in equilibrium with an infinite reservoir of black-body radiation. In the absence of boundary conditions the stress tensor at infinity is equal to

$$< H|T^\mu_\nu|H >= \frac{\pi T^2}{12}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}. \tag{28}$$

In the presence of the boundary conditions, equation (22) has to be added to the above relation. In this case at $r \to \infty$ we have

$$< H|T^\mu_\nu|H >= (\frac{1}{384\pi m^2} + \varepsilon_c^1)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{29}$$

then

$$\beta = \varepsilon_c^1 - \frac{1}{192\pi m^2}, \qquad \alpha = 0. \tag{30}$$

The difference between (30)and (26) is due to the existence of the bath of thermal radiation at temperature T.

In order to calculate the contribution from the Hawking evaporation process to the Casimir energy(total vacuum energy), for this special geometry. We introduce the final quantum state which is a convenient candidate for the vacuum [41]. This state is called the Unruh state $|U>$. In the limit $r \to \infty$, this state corresponds to the outgoing flux of a black-body radiation at black hole temperature T. The stress tensor in the limit $r \to \infty$ and in the absence of the boundary conditions is as follows:

$$< U|T^\mu_\nu|U >= \frac{\pi T^2}{12}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}. \tag{31}$$

In the presence of the boundary conditions the expression (22) should be added to the relation (31) which is a new stress tensor. Comparing the new stress with Eq.(20) one obtains

$$\alpha = \frac{-1}{768\pi m^2}, \qquad \beta = \varepsilon_c^1 - \frac{1}{256\pi m^2}. \tag{32}$$

Finally, the form of the stress tensor(20), in, respectively Boulware ,Hartle-Hawking and Unruh states is as follows:

$$< B|T^\mu_\nu(r)|B >= \begin{pmatrix} T^\alpha_\alpha - \Omega^{-1}(r)H(r) & 0 \\ 0 & \Omega^{-1}H(r) \end{pmatrix}$$
$$+\Omega^{-1}(\frac{J_1(B_1, B_2)}{2\pi^{1/2}a^2\Gamma(3/2)} + \frac{1}{384\pi m^2})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{33}$$

$$< H|T^{\mu}_{\nu}(r)|H >= \begin{pmatrix} T^{\alpha}_{\alpha} - \Omega^{-1}(r)H(r) & 0 \\ 0 & \Omega^{-1}H(r) \end{pmatrix}$$

$$+\Omega^{-1}\left(\frac{J_1(B_1,B_2)}{2\pi^{1/2}a^2\Gamma(3/2)} + \frac{1}{192\pi m^2}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (34)$$

$$< U|T^{\mu}_{\nu}(r)|U >= \begin{pmatrix} T^{\alpha}_{\alpha} - \Omega^{-1}(r)H(r) & 0 \\ 0 & \Omega^{-1}H(r) \end{pmatrix}$$

$$+\Omega^{-1}\begin{pmatrix} \frac{J_1(B_1,B_2)}{2\pi^{1/2}a^2\Gamma(3/2)} + \frac{1}{256\pi m^2} & \frac{-1}{768\pi m^2} \\ \frac{1}{768\pi m^2} & -\frac{J_1(B_1,B_2)}{2\pi^{1/2}a^2\Gamma(3/2)} - \frac{1}{256\pi m^2} \end{pmatrix}. \quad (35)$$

Here the presence of the form

$$\frac{J_1(B_1,B_2)}{2\pi^{1/2}a^2\Gamma(3/2)}\Omega^{-1}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (36)$$

is due to the boundary conditions. Therefore, Eqs.(33-35)are separable as follows:

$$< B|T^{\mu}_{\nu}|B >=< B|T^{(g)\mu}_{\nu}|B > + < B|T^{(b)\mu}_{\nu}|B > \quad (37)$$

$$< H|T^{\mu}_{\nu}|H >=< H|T^{(g)\mu}_{\nu}|H > + < H|T^{(b)\mu}_{\nu}|H > + < H|T^{(t)\mu}_{\nu}|H > \quad (38)$$

$$< U|T^{\mu}_{\nu}|U >=< U|T^{(g)\mu}_{\nu}|U > + < U|T^{(b)\mu}_{\nu}|U > + < U|T^{(r)\mu}_{\nu}|U > \quad (39)$$

where $< T^{(g)\mu}_{\nu} >$, $< T^{(b)\mu}_{\nu} >$ and $< T^{(r)\mu}_{\nu} >$ correspond to gravitational, boundary and Hawking radiation contributions, respectively, and $< T^{(t)\mu}_{\nu} >$ is the bath of thermal radiation at temperature T. It should be noted that the trace anomaly has a contribution just in the first term $< T^{(g)\mu}_{\nu} >$, which comes from the background effect not the boundary one. However, it has a contribution in the total Casimir energy-momentum tensor. In the regions $r < r_1$and $r > r_2$ the boundary parts are zero and only the gravitational polarization parts are present.
The vacuum boundary part pressures acting on plates are

$$P_{b_{1,2}} = P_b(r = r_{1,2}) = - < T^{(b)1}_{1}(r = r_{1,2}) >= \Omega^{-1}(r_{1,2})\frac{J_1(B_1,B_2)}{2\pi^{1/2}a^2\Gamma(3/2)}, \quad (40)$$

this corresponds to the attractive/repulsive force between the plates if $P_{b_{1,2}} < / > 0$. The equilibrium points for the plates correspond to the zero values of Eq.(40): $P_{b_{1,2}} = 0$. These points are zeros of the function $J_1(B_1, B_2)$ defined by Eq.(23) and are the same for both plates. The effective pressure created by other parts in (33-35) are the same for both sides of the plates, and hence lead to the vanishing effective force.

4 Conclusion

In the semiclassical approximation theory of quantum gravity we are involved in calculating the expectation value of energy-momentum tensor in special vacuum [8]. However, the usual expression for the stress tensor includes singular products of the field operators for stress tensor. Renormalization theory of the stress tensor claims to solve this problem, but it must be mentioned that the usual scheme of renormalization includes complexity

and somewhat ambiguity. For instance, there is no conceptual support for a local measure of energy-momentum of some given state without any reference to any global structure. We know in this frame energy is source of gravity and we are not allowed to subtract any unwanted part of energy even though it is infinite. So to consider the back-reaction effect of the quantum field on the gravitational field, we must find a more elaborate renormalization scheme in which the dynamics of gravitational field is a vital component. In the present paper we have found the renormalized energy-momentum tensor for a massless scalar field on background of two dimensional Schwarzschild black hole for two plates with Robin boundary conditions, by making use of general properties of stress tensor only. We propose that if we know the stress tensor for a given boundary in Minkowski space-time, the Casimir effect in gravitational background can be calculated. We have found direct relation between trace anomaly and total Casimir energy. In addition, by considering the Hawking radiation for observer far from black hole, this radiation contributes to the Casimir effect.

In this paper we have derived three renormalized energy-momentum tensors for our case under study. This is due to selecting three types of vacuum states for our calculation. If we consider the Boulware vacuum, the stress tensor will have two parts: a boundary part and a gravitational part. However, using Hartle-Hawking and Unruh vacuums will result in another term being added to the stress tensor, which, respectively, corresponds to a bath of thermal radiation and Hawking radiation. In the region between the plates the boundary induced part for the vacuum energy-momentum tensor is given by Eq.(36), and the corresponding vacuum forces acting on the plates have the form Eq.(40). These forces vanish at the zeros of the function $J_1(B_1, B_2)$. For a conformally coupled massless scalar field with Robin boundary condition this effect was initially studied in Ref.[42] for a background Randall–Sundrum geometry [43]. Therefore in this case the Casimir effect provide a possibility for the stabilization of the distance (radion field) between the branes (for more study see [44]). The effective pressure created by other parts in Eqs.(33, 34, 35) are the same from the both sides on the plates, and hence leads to the zero effective force.

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